A stable, efficient, locking free hexahedral element for problems in non-linear dynamics

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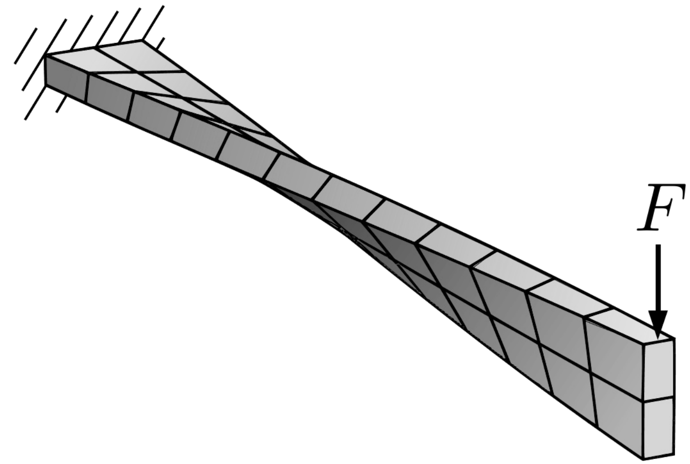
Motivation and Goals

- Low order hex elements perform poorly for:
  - Bending dominated problems
    - Exhibit shear locking
  - Nearly incompressible materials
    - Exhibit volumetric locking
- Seek hex elements which are:
  - General purpose
    - Can represent solid or shell structures
    - Suitable for finite deformations
    - Compatible with most material models
  - Efficient
  - Locking free
  - Stable



	Tip displacement
Exact solution	1.000
Hex 8	0.206
“B-Bar” Hex 8	0.232

Contemporary Approaches…

- Enhanced assumed strain (EAS) methods (Simo & Rifai, 1990)
- Physically stabilized elements with reduced integration (Puso, 2000)
- Assumed natural strain (ANS) methods (Radovitzky & Dvorkin, 1994)
- Solid/thick shell formulations (Hughes & Liu, 1981)

…and Their Shortcomings

- Enhanced assumed strain (EAS) methods (Simo & Rifai, 1990)
  - Must iteratively solve for the enhanced variables (slow)
  - Suffers from numerical instabilities; requires artificial stabilization
- Physically stabilized elements with reduced integration (Puso, 2000)
  - Poorly resolved plastic bending response in coarse meshes
  - Adaptively adjusting the stabilization parameters to better represent plastic bending can result in unphysical energy growth
- Assumed natural strain (ANS) methods (Radovitzky & Dvorkin, 1994)
  - Not compatible with general (rate-formulated) constitutive models
  - Suffers from numerical instabilities; requires artificial stabilization
- Solid/thick shell formulations (Hughes & Liu, 1981)
  - Not a general purpose element (only intended for modeling shell structures)
  - Not compatible with most continuum constitutive models

Mixed-Enhanced Strain (MES) Elements

- Mixed-enhanced strain approach (Kasper & Taylor, 2000)
- Formulation derived from a 3-field Hu-Washizu functional:

$$\Pi^{\mathrm{int}} \equiv \int_{\Omega_0} W(\mathbf{F})\, dV + \int_{\Omega_0} \mathbf{P} : [\nabla \mathbf{x} - \mathbf{F}]\, dV$$

- Shear locking is eliminated via a "strain projection" procedure
  - Similar to an ANS/mixed method
  - Shear enhancement terms determined directly (require no iteration)
- Volumetric locking is eliminated through the addition of enhanced fields
  - Similar to an EAS method
  - Must iteratively solve for the volumetric enhancement terms

Weak Enforcement of the Volume Constraint

- **Novelty**: add the following term to the Hu-Washizu functional:

$$\frac{1}{3} \int_{\Omega_0} \mathrm{tr}(\boldsymbol{\tau}^*) \; [\log(\det(\nabla \mathbf{x})) + \mathrm{tr}(\mathbf{H}^*)] \; dV$$

- Weakly enforces the volume-preserving constraint ($\det \mathbf{F} = 1$) against enhanced fields in the setting of finite deformations
- Similar to approach proposed by (Simo, Taylor, & Pister, 1984), but more general (uses tensor-valued enhancements)
- Choose enhanced fields to eliminate volumetric locking, while preserving the effects of anticlastic curvature in bending
  - Scalar-valued enhancements are *not* sufficient to this end
  - Tensor-valued enhancements are needed

A Modified Mixed-Enhanced Strain Approach

- Modified Hu-Washizu variational principle:

$$\Pi^{\text{int}} \equiv \int_{\Omega_0} W(\mathbf{F}^{\dagger})\, dV + \int_{\Omega_0} \mathbf{P} : [\nabla \mathbf{x} - \mathbf{F}]\, dV$$
$$+ \frac{1}{3} \int_{\Omega_0} \operatorname{tr}(\boldsymbol{\tau}^*)\ [\log(\det(\nabla \mathbf{x})) + \operatorname{tr}(\mathbf{H}^*)]\, dV$$

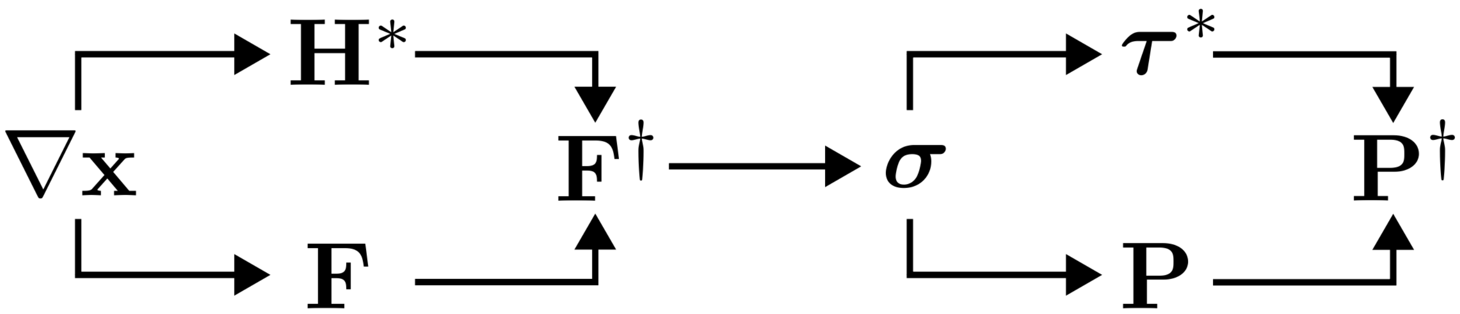
- Define the modified deformation gradient as:

$$\mathbf{F}^{\dagger} = \alpha \mathbf{F}^* \mathbf{F} \qquad \alpha = \sqrt[3]{\frac{\det(\nabla \mathbf{x})}{\det(\mathbf{F})}} \qquad \mathbf{F}^* = \exp(\mathbf{H}^*)$$

- Combines both shear and volumetric enhancements

Strain and Stress Projection Operations

- Sequence of (linear) projection operators fully determine the enhanced strain and stress variables:



- Element internal forces integrated using the modified first P-K stress:

$$\mathbf{f}_a^{\mathrm{int}} = \int_{\Omega_0} \mathbf{P}^\dagger \cdot \nabla \varphi_a \, dV$$

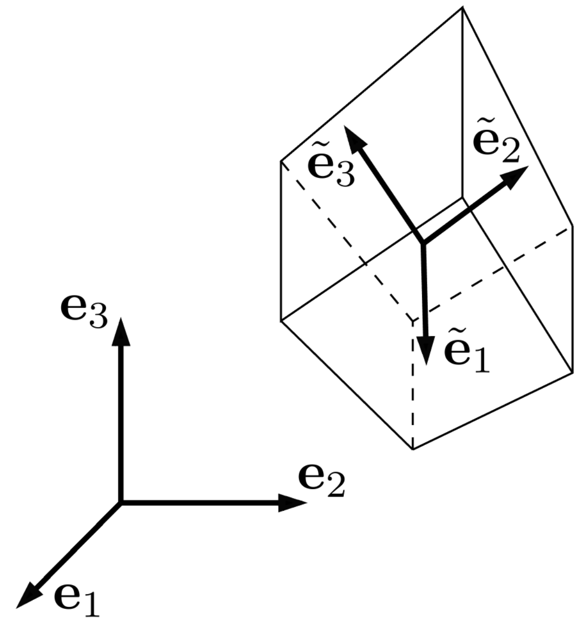
- **Requires no non-linear iteration at the element level to solve for the enhanced fields!**

Frame Invariance

- Establish an element coordinate frame corresponding to the oblique transformation defined by the element's Jacobian:

$$\tilde{\mathbf{e}}_i = \bar{\mathbf{J}} \cdot \mathbf{e}_i$$

$$\bar{\mathbf{J}} \equiv \left. \frac{\partial \mathbf{x}}{\partial \boldsymbol{\xi}} \right|_{\boldsymbol{\xi}=\mathbf{0}}$$



- Define all enhanced fields within this frame (i.e. in parent element coordinates) to maintain frame invariance

Patch Test Satisfaction

- Satisfaction of patch tests is achieved by ensuring that the enhancements are L_2 orthogonal to a constant field:

$$\int_{\Omega_0} \hat{\mathbf{F}}\, dV = \mathbf{0} \qquad (\mathbf{F} = \nabla \mathbf{x} + \hat{\mathbf{F}})$$

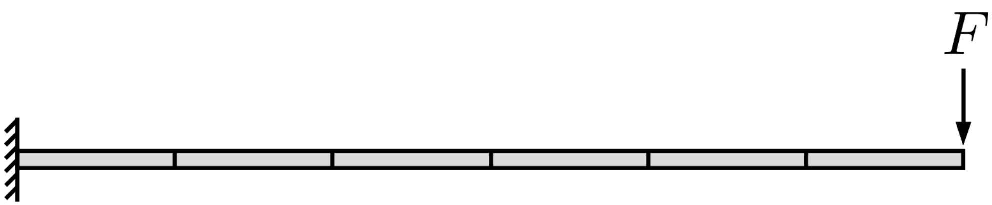
$$\int_{\Omega_0} \mathrm{tr}(\mathbf{H}^*)\, dV = 0$$

- Select shear and volumetric enhancements which satisfy the above conditions a priori, resembling (Glaser & Armero, 1997):

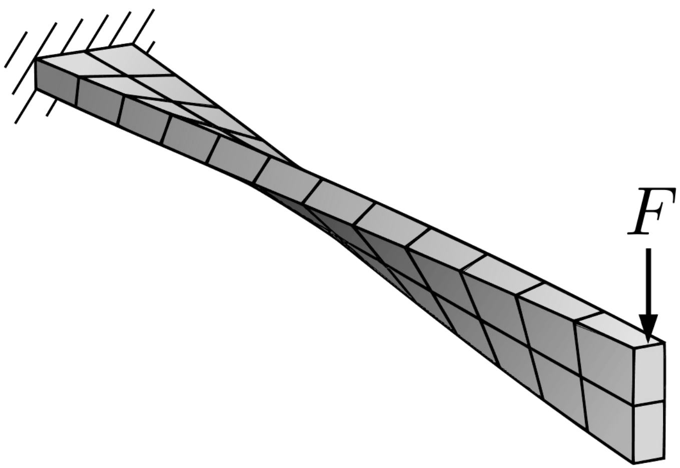
$$\hat{\mathbf{F}} = \begin{bmatrix} 0 & \hat{F}_{12}\xi & \hat{F}_{13}\xi \\ \hat{F}_{21}\eta & 0 & \hat{F}_{23}\eta \\ \hat{F}_{31}\zeta & \hat{F}_{32}\zeta & 0 \end{bmatrix} \qquad \mathbf{H}^* = \begin{bmatrix} h_1^*\xi & 0 & 0 \\ 0 & h_2^*\eta & 0 \\ 0 & 0 & h_3^*\zeta \end{bmatrix}$$

Performance in Bending

- Good coarse mesh accuracy for benchmark bending problems



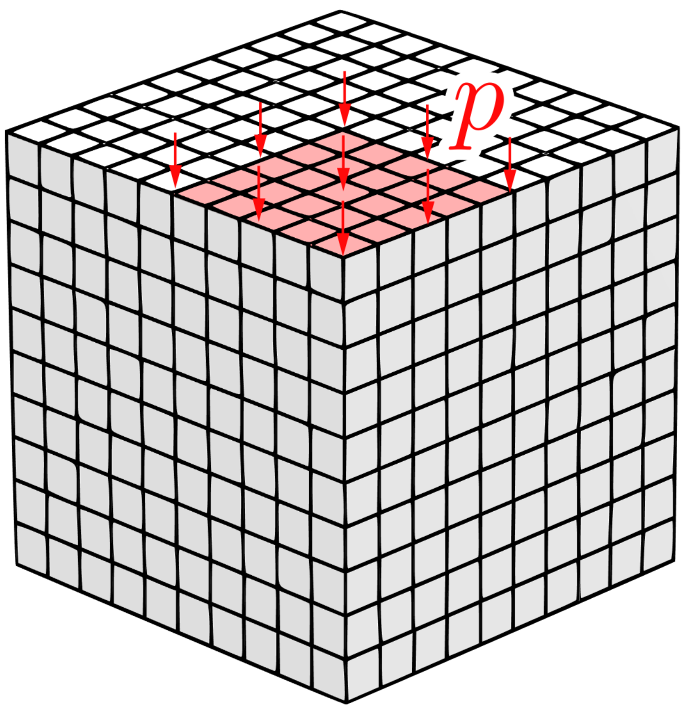
	Tip displacement
Exact solution	1.000
Hex 8	0.025
"B-Bar" Hex 8	0.026
MES Hex 8	0.999

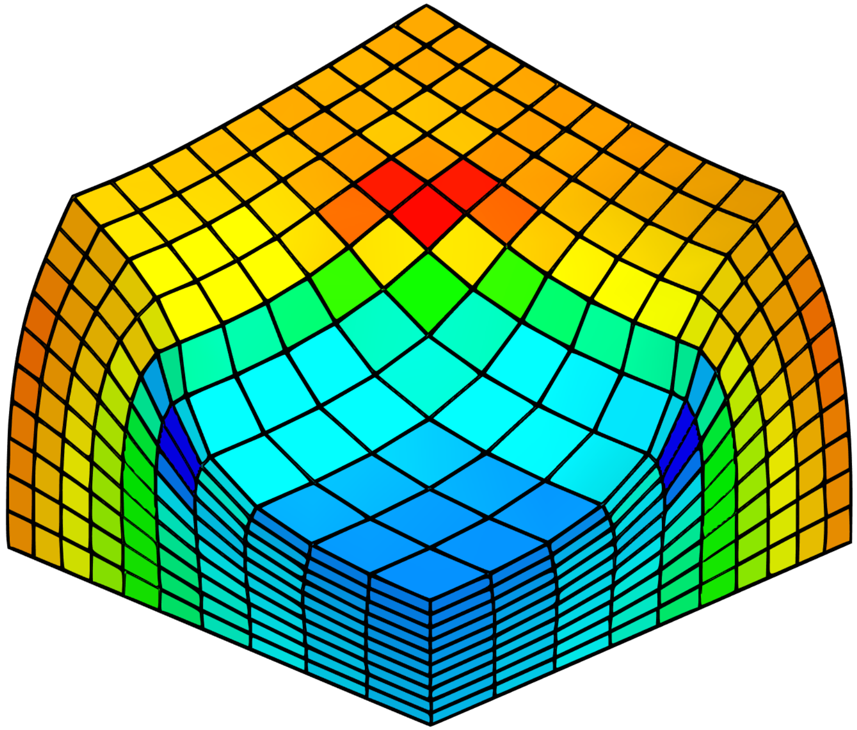


	Tip displacement
Exact solution	1.000
Hex 8	0.206
"B-Bar" Hex 8	0.232
MES Hex 8	0.941

Performance in Nearly Incompressible Problems

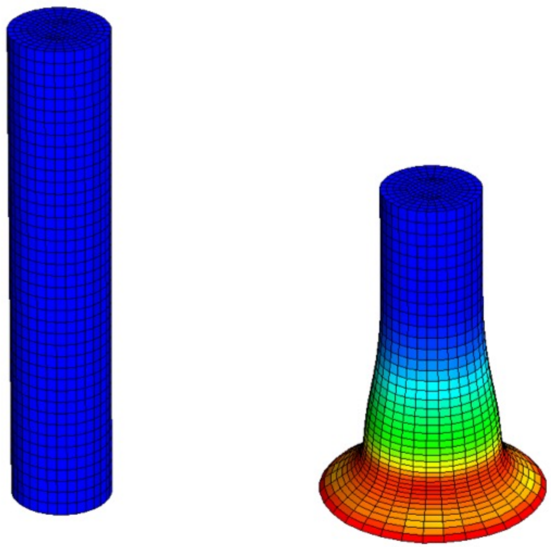
- Reduced volumetric locking (some mild checkerboarding)
- No apparent instabilities for highly compressive deformations

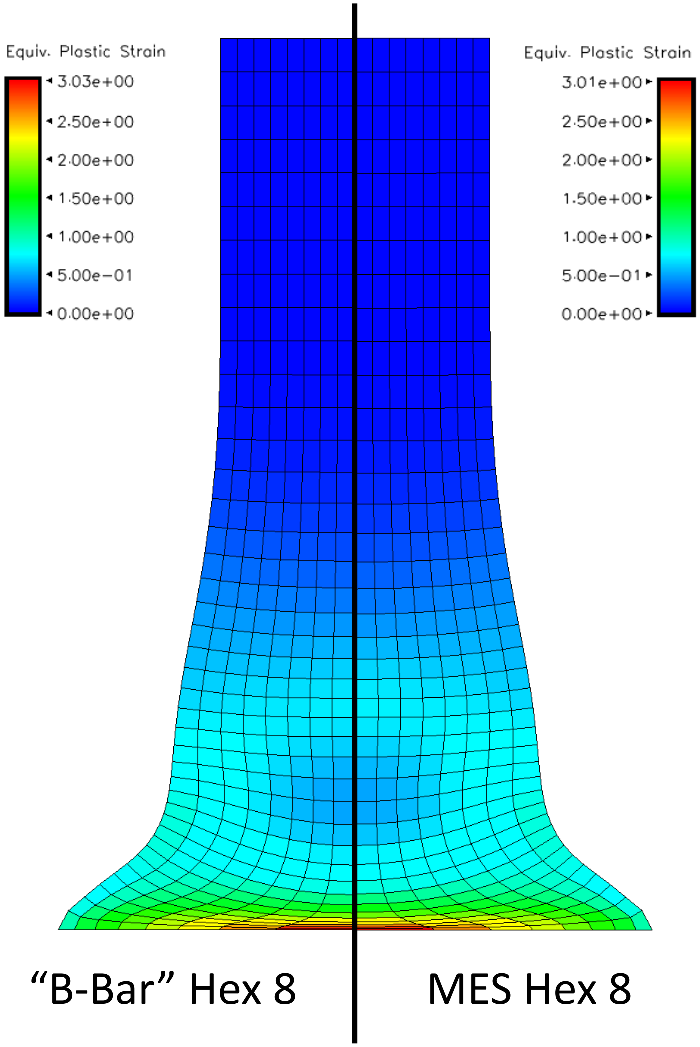




Performance in Problems with Plasticity

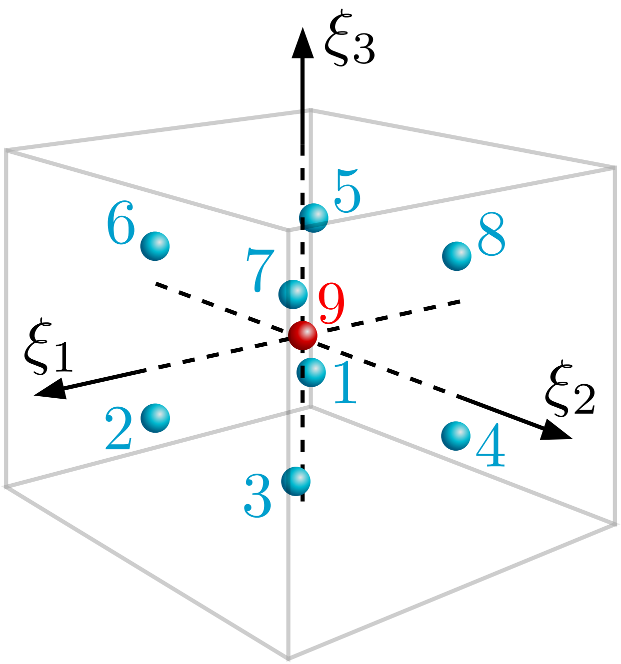
- Taylor bar impact problem:
  - No volumetric locking
  - No apparent instabilities at high plastic strain rates
  - Results indistinguishable from standard “B-Bar” Hex 8

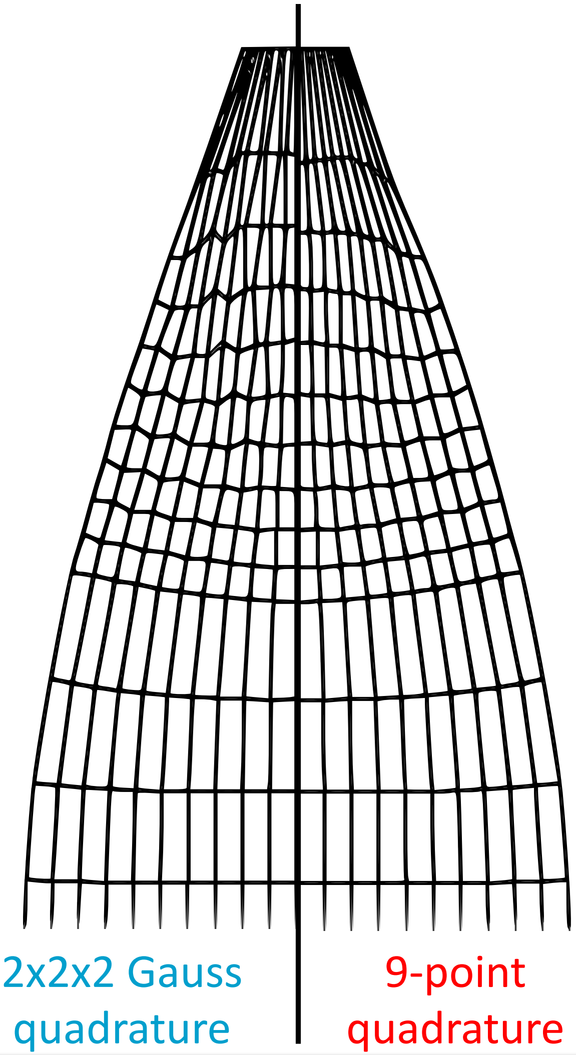




Performance in Elasto-Plastic Necking Problems

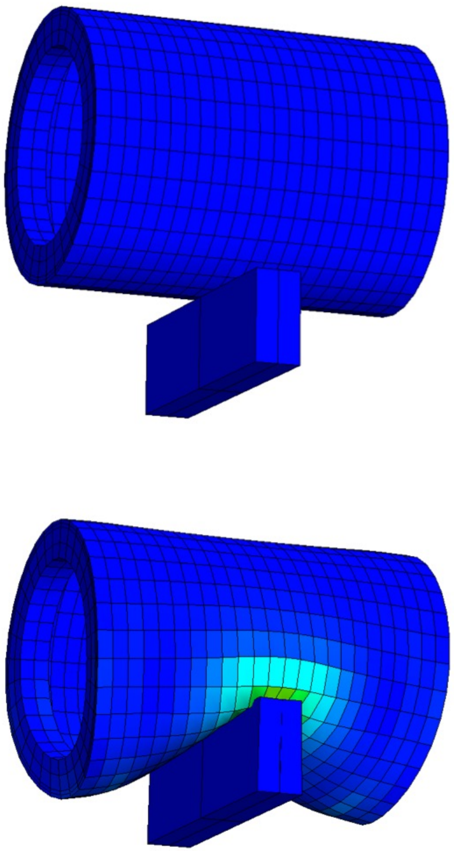
- Modified 9-point quadrature rule (Simo, Armero, & Taylor, 1993) reduces hourglassing instabilities in elasto-plastic necking problems

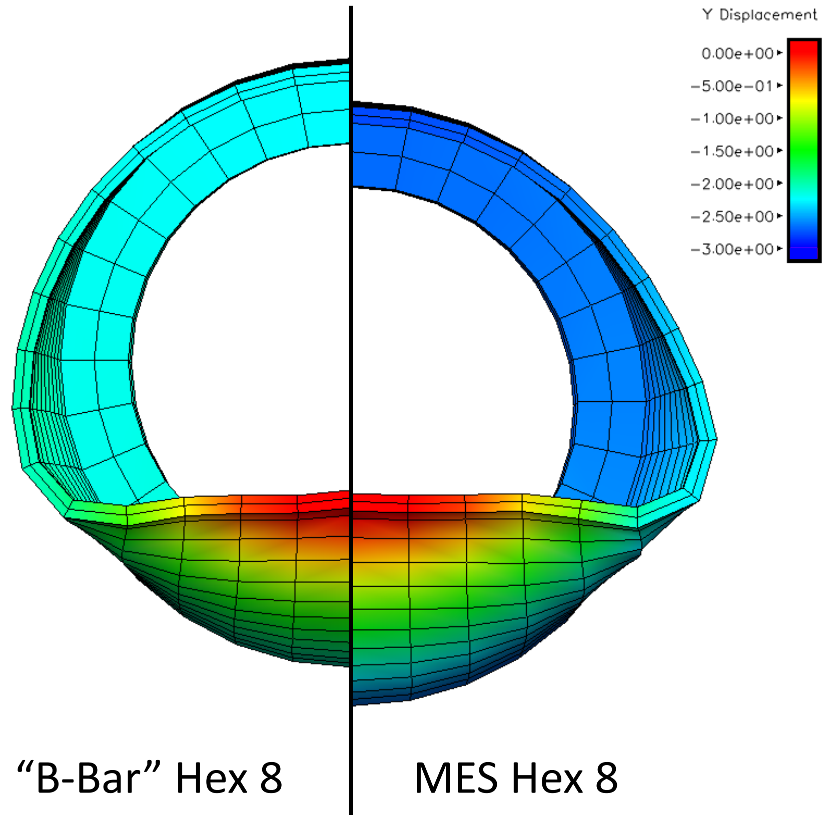




Cylinder Impacting a Rail

- Captures localized plastic bending of thin shell-like structures:





Conclusions and Future Work

- Chosen approach yields comparable performance with EAS methods, while circumventing the need for non-linear iteration at the element-level to obtain enhancements
- 9-point quadrature scheme reduces hourglassing for tensile necking problems
- May nonetheless exhibit instabilities for sufficiently distorted elements undergoing severe plastic deformations
  - Currently exploring various means of physically stabilizing the element

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