

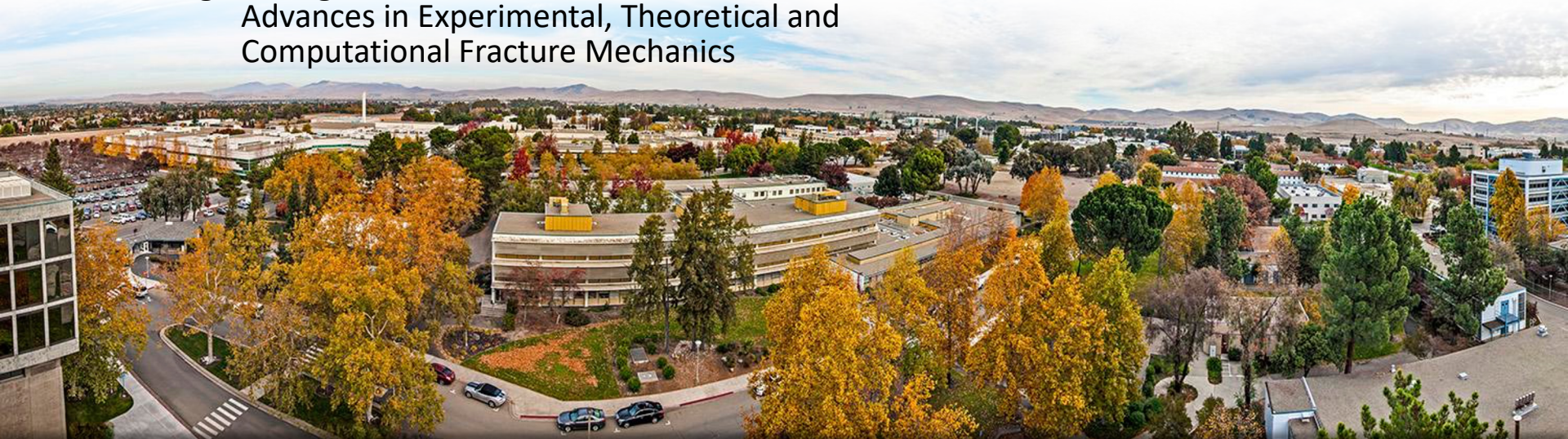
# A Smeared Crack Modeling Framework Accommodating Multi-directional Fracture at Finite Strains

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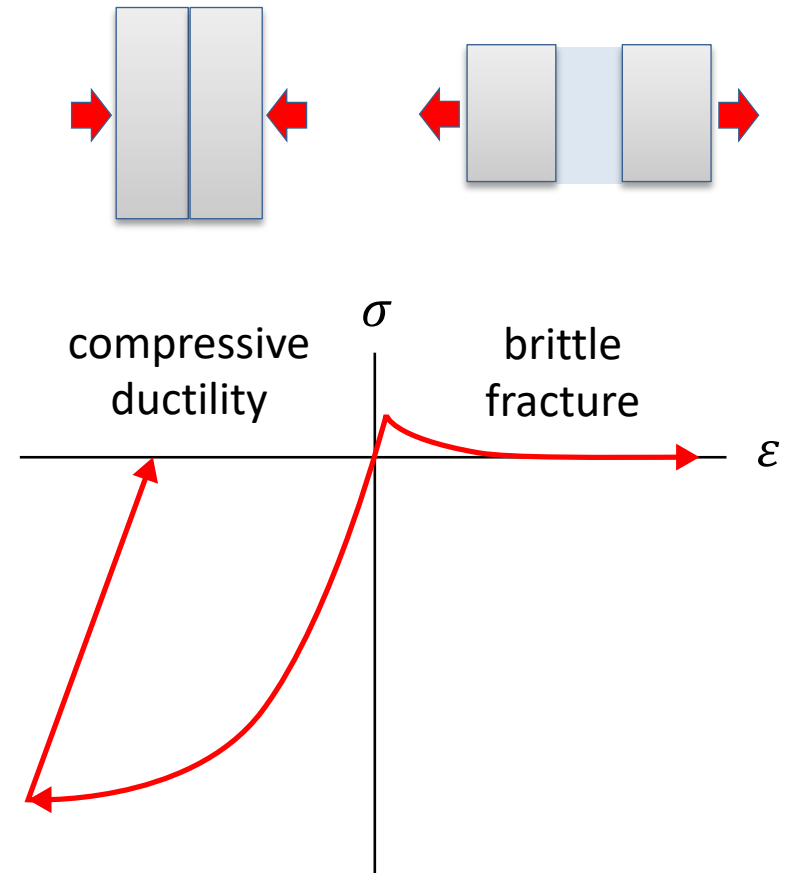
Advances in Experimental, Theoretical and  
Computational Fracture Mechanics

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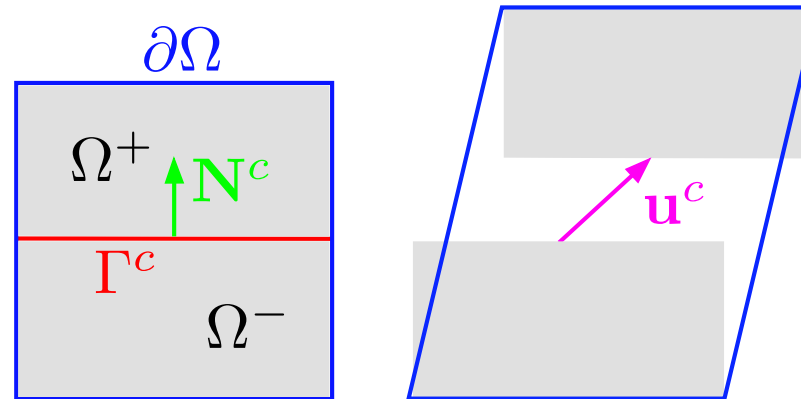


# Motivation and Goals

- Modeling of fracture for large-scale explicit dynamics problems on HPC platforms
- Permit multi-directional cracking to robustly resolve branching
- Desire a “classical” smeared crack modeling approach
  - More easily implemented in finite element codes in the format of a constitutive idealization
- Desire a model which supports brittle fracture in combination with more complicated material behavior (plasticity, visco-elasticity, etc.)
  - Intended for modeling hot isostatically pressed (HIP) metal powder composites



# Embedded cohesive interface concept, based upon the “DGD” approach of Leone (2016)\*

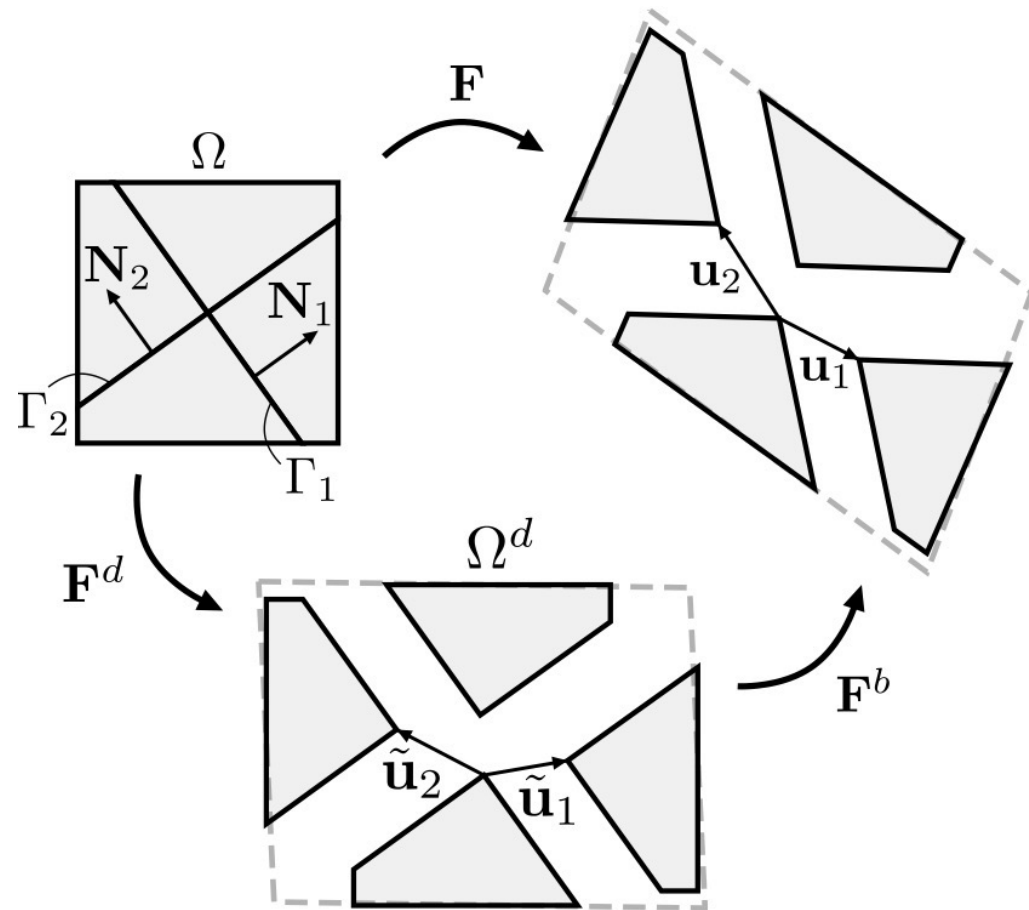


- Cracks are modeled as embedded strong discontinuities within a surrounding “bulk” constitutive model
  - SCM extension to finite strains, permitting interfacial “slip”
- Damage evolved by a cohesive traction-separation law for  $\mathbf{u}^c$
- Crack opening is governed by crack-interface traction equilibrium
  - Recompression is accommodated via internal contact constraints

\*F. Leone. Deformation gradient tensor decomposition for representing matrix cracks in fiber-reinforced materials. Composites Part A Applied Science and Manufacturing, 76:334{341, 09 2015. doi: 10.1016/j.compositesa.2015.06.014.

# Multiplicative split of the deformation gradient into “damage” and “bulk” material motions

- DGD approach of Leone extended to accommodate up to 3 mutually orthogonal cracks
  - Multiple cracks required to manage branching
- New crack planes initiated by critical stress-based interface condition
  - Failure criterion induced by choice of embedded cohesive model (e.g. max prin. stress)



# Compatible with any generic choices for the “cohesive” and “bulk” material responses

- Formulated within a generic thermodynamic framework:

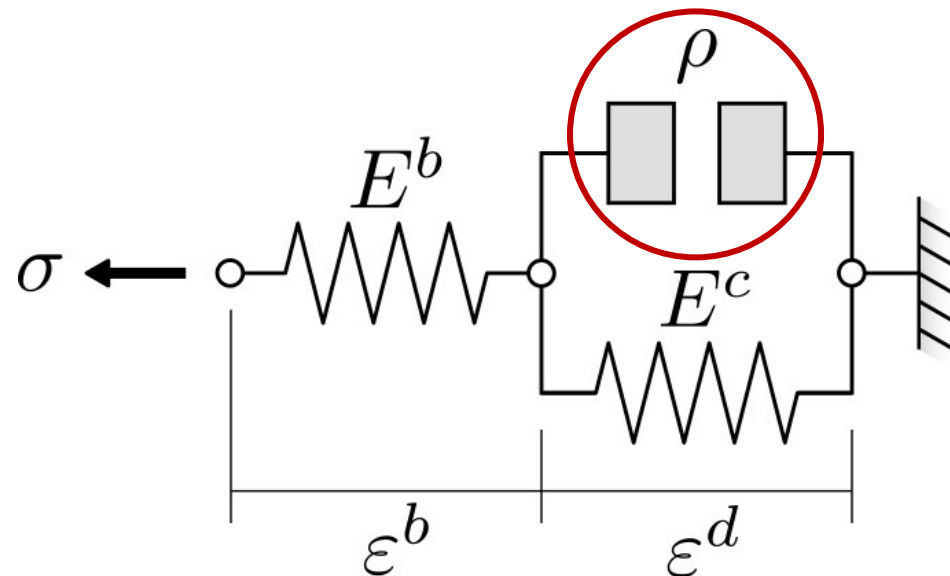
$$\psi = \psi^d(\mathbf{F}^d) + \psi^b(\mathbf{F}^b)$$

- Multiplicative decomposition of the deformation gradient into three parts:  $\mathbf{F} = \mathbf{F}^e \mathbf{F}^p \mathbf{F}^d$ 
  - Damage:  $\mathbf{F}^d$
  - Plasticity:  $\mathbf{F}^p$
  - Elasticity:  $\mathbf{F}^e$

    } Bulk:  $\mathbf{F}^b = \mathbf{F}^e \mathbf{F}^p$
- Energetic consistency achieved through separable damage and bulk (plasticity) dissipation mechanisms

# Dynamic regularization: 1D conceptual idealization of an “inertial stress element”

- Encapsulate the effects of dynamic crack opening within the *constitutive model*
- Postulate “internal kinetic energy” associated with the damage deformation process
  - Induces “inertial stress” due to crack opening
- Tantamount to a dynamic (hyperbolic) regularization of the interface traction equilibrium equations
  - Update  $\varepsilon^d$  *explicitly* in time



$$\psi = \frac{1}{2}E^b(\varepsilon^b)^2 + \frac{1}{2}E^c(\varepsilon^d)^2 + \underline{\frac{1}{2}m(\dot{\varepsilon}^d)^2}$$

$$\sigma = E^b\varepsilon^b = E^c\varepsilon^d + \underline{m\ddot{\varepsilon}^d}$$



# Extension to 3D: idealize material RVE as an ellipsoidal domain $\Omega$ with finite extent

- Express *internal kinetic energy* w.r.t.  $\mathbf{G}$  and  $\dot{\mathbf{F}}^d$ :

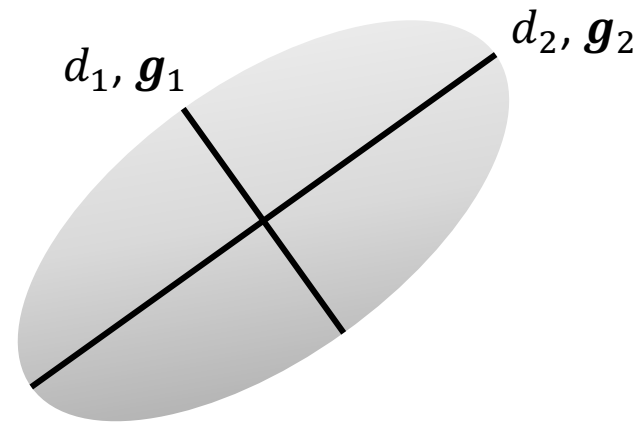
$$T^d(\dot{\mathbf{F}}^d) = \frac{2\rho}{9} \text{tr} \left( \dot{\mathbf{F}}^d \mathbf{G} (\dot{\mathbf{F}}^d)^T \right)$$

- Characteristic fracture length scale  $\ell_c$  for a given crack with normal  $\mathbf{N}_c$  computed via metric-induced norm:

$$\ell_c = \frac{4}{3} \|\mathbf{N}_c\|_{\mathbf{G}}$$

- Postulate homogenized RVE energy, including *internal KE*:

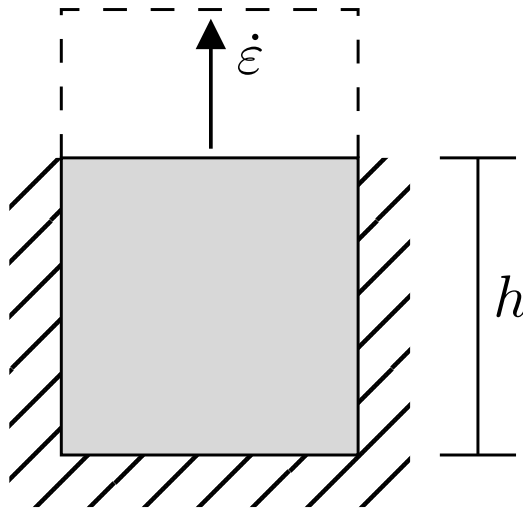
$$\int_{\Omega} \bar{\psi} dV = \int_{\Omega} \psi^b dV + \sum_{c=1}^{N_{\text{cracks}}} \int_{\Gamma_c} \Psi_c dA + \int_{\Omega} T^d dV.$$



- Define element size-dependent RVE inertia tensor  $\mathbf{G}$ :

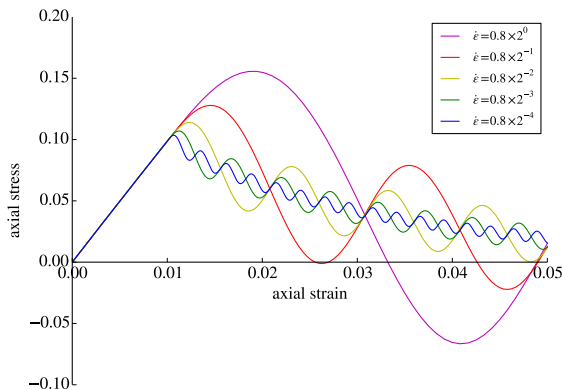
$$\mathbf{G} = \sum_{i=1}^3 \left( \frac{d_i}{2} \right)^2 \mathbf{g}_i \otimes \mathbf{g}_i$$

# Single-element demonstration of uniaxial model behavior at a constant loading rate

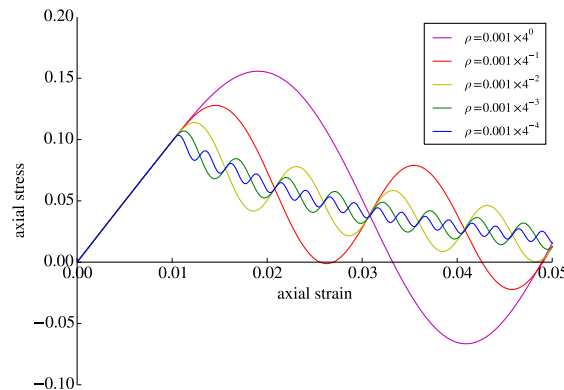


- Dynamic regularization conserves energy but induces transient oscillations
- Behavior approaches quasi-static response under the following limiting conditions, shown below:

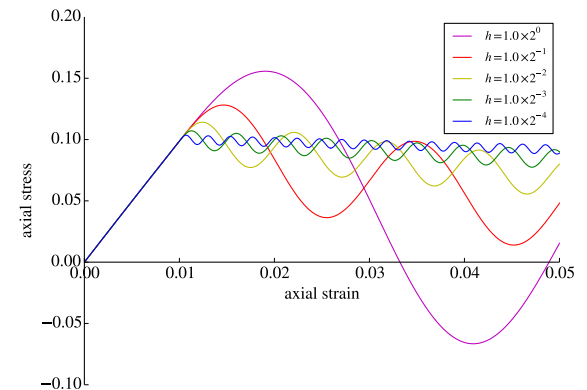
strain rate  $\dot{\epsilon} \rightarrow 0$



density  $\rho \rightarrow 0$

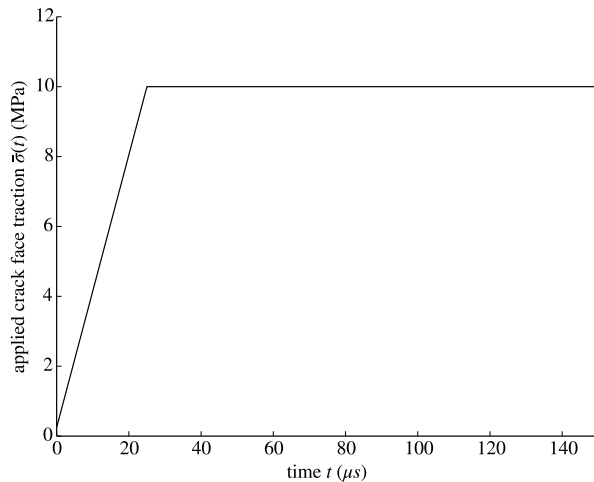
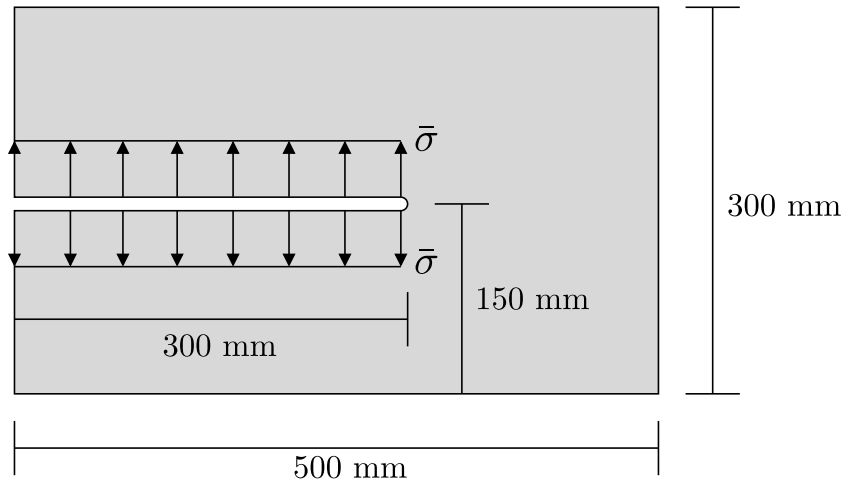


element size  $h \rightarrow 0$

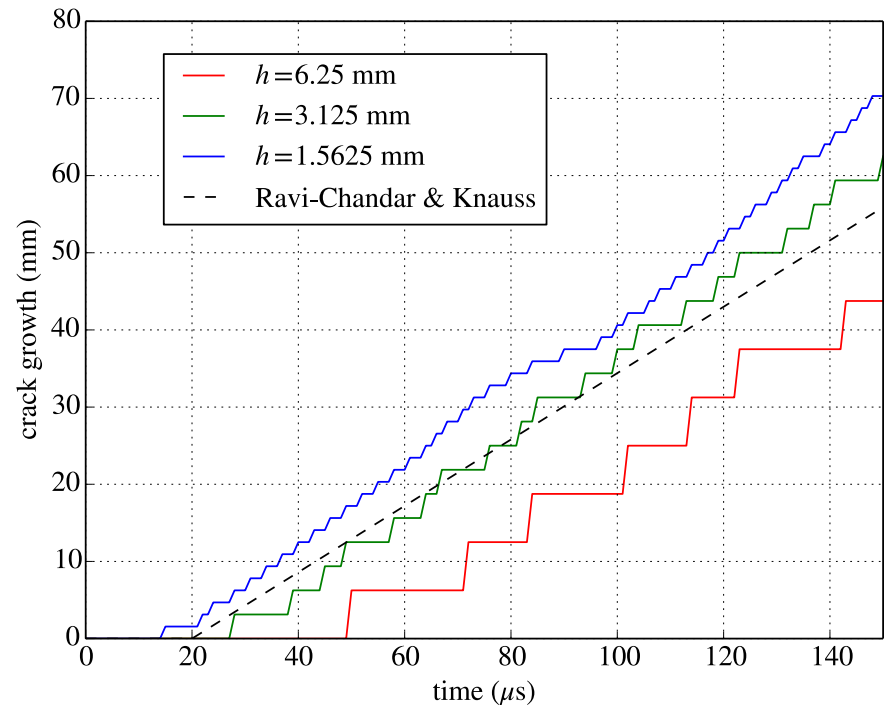




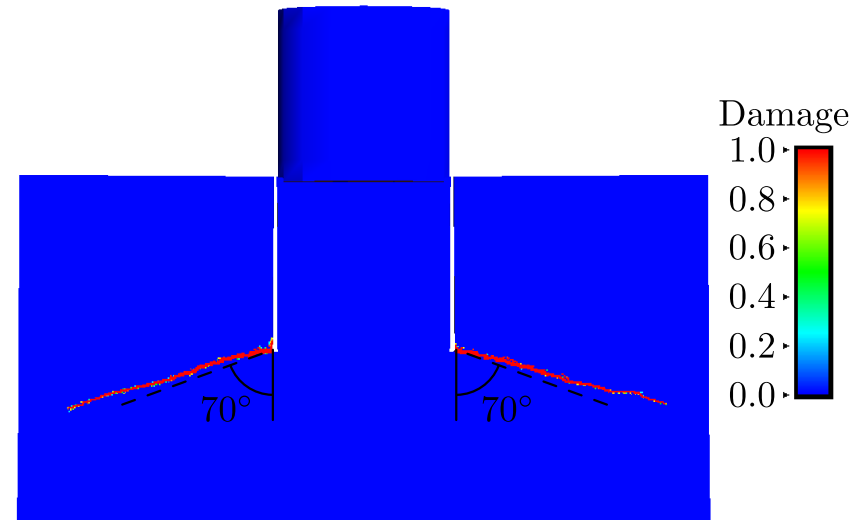
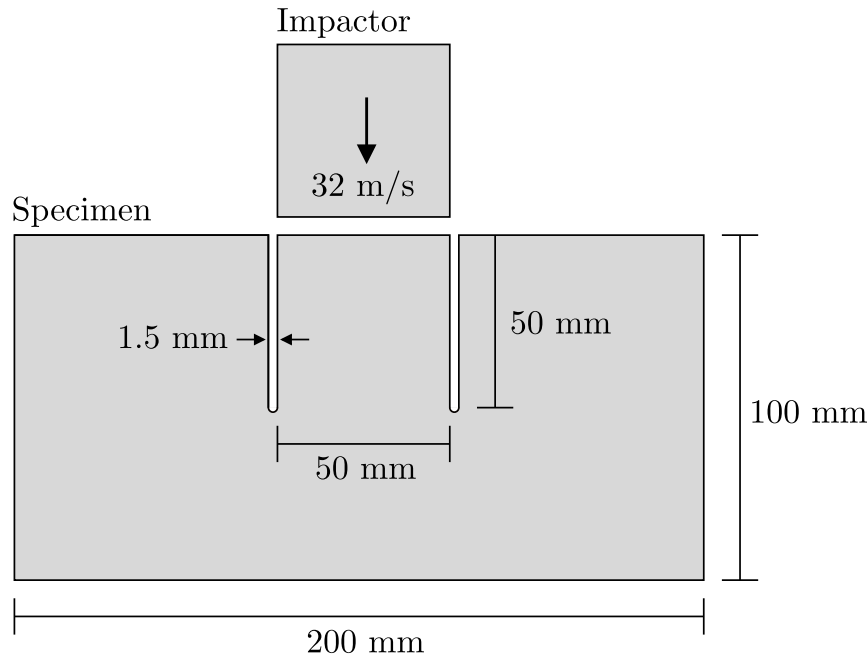
# Examination of crack growth rate: validation against experiment of Ravi-Chandar & Knauss



	Average crack growth rate (m/s)	Crack initiation time ( $\mu s$ )
$h = 6.25$ mm	421	50
$h = 3.125$ mm	503	28
$h = 1.5625$ mm	510	15
Ravi-Chandar & Knauss	430	20



# Influence of mesh design on crack path: validation against Kalthoff-Winkler experiment



- Local formulation of material damage at the constitutive level still induces the effects of mesh bias
  - Well-known problem with SCM approaches

# Conclusions and Future Work

- Current implementation of the model demonstrates viability of the dynamic regularization concept
  - Fully decouples “bulk” and “cohesive” material update procedures
  - Efficient constitutive update (beneficial for use in explicit dynamics)
- “Local” form results in mesh bias, influence on crack path
- Future work will seek a non-local regularization, or adoption of a phase field approach to mitigate the effects of mesh bias on crack path



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