Elasto-Plastic Hourglass Control for Physically Stabilized Non-Linear Finite Elements with Reduced Integration

Brian Giffin
Methods Development Group
Computational Engineering Division
Lawrence Livermore National Laboratory

June 2, 2022

Engineering Mechanics Institute Conference 2022

Computational Methods and Applications for Solid and Structural Mechanics

LLNL-PRES-835482

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

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Outline

1. Review of “*hourglass*” (HG) control & the need for improvements

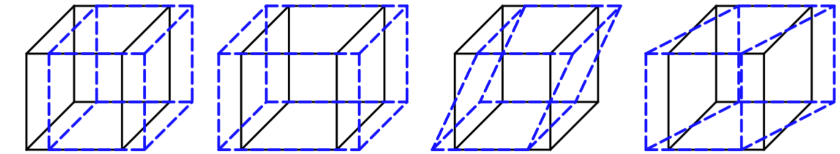
2. Theoretical framework of the “*physical stabilization*” (PS) hourglass control scheme [1]

3. Extend PS method to accommodate plastic deformations

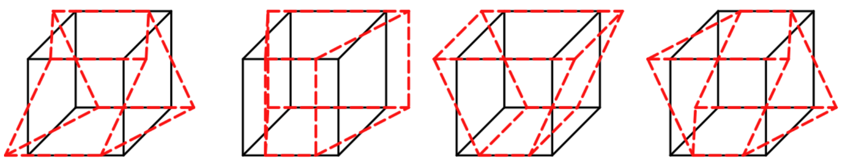
4. Example demonstration of the method

[1] M. A. Puso. A highly efficient enhanced assumed strain physically stabilized hexahedral element. International Journal for Numerical Methods in Engineering, 49(8):1029-1064, 2000.

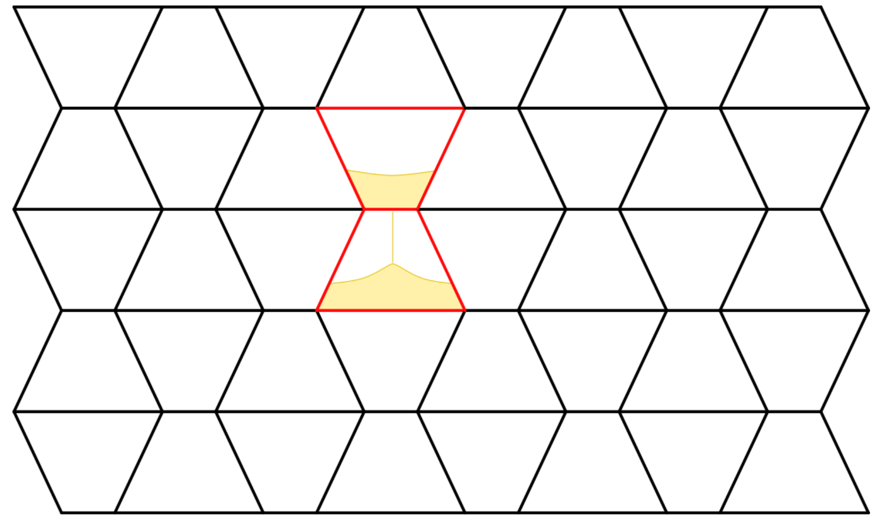
What is hourglass (HG) control, and why do we need it?



Deformation modes with a uniform strain distribution
(representable using 1-point integration)

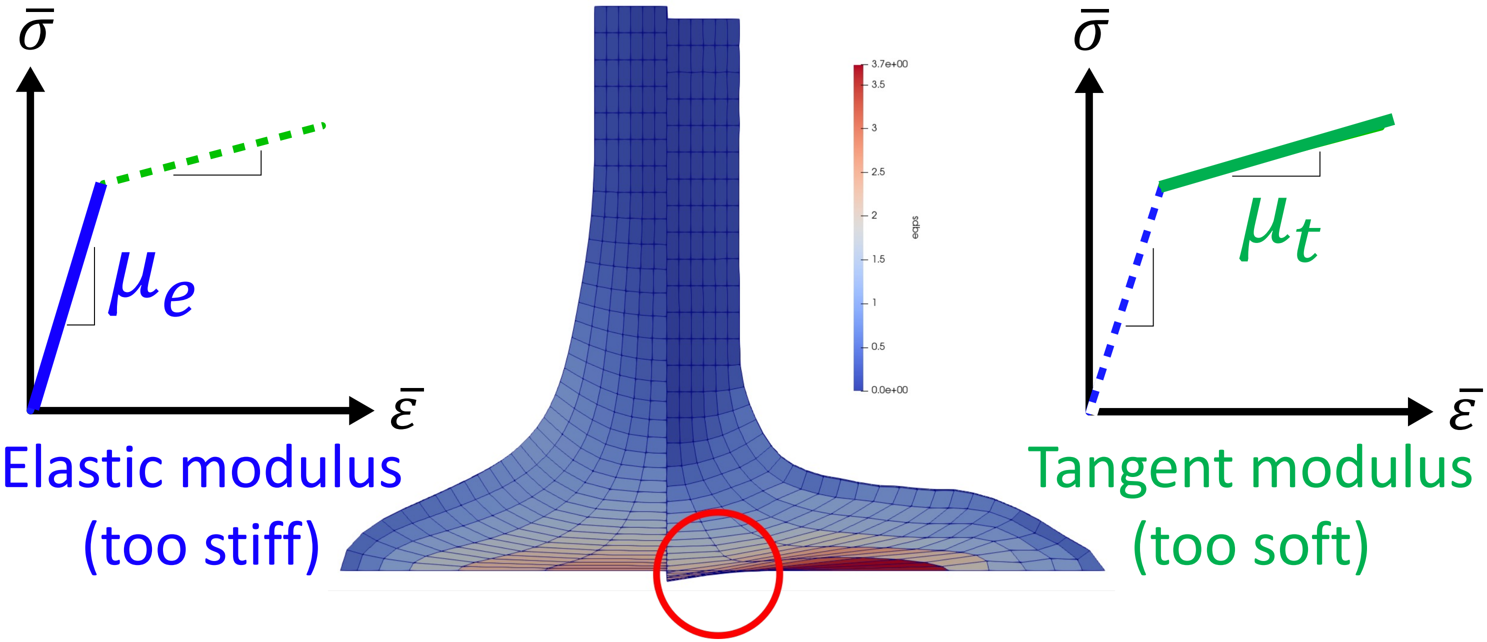


Deformation modes with a non-uniform strain distribution
(modes with zero internal energy using 1-point integration)



- Reduced (1-point) integration schemes for low-order elements only sample the constant strain modes of deformation in an element
- Lack of stabilization results in unphysical hourglass deformations
- **Hourglass control** = (artificial) stabilization to resist hourglass modes

Hourglass control for non-linear materials relies on heuristic choice of stabilization modulus



- Large deformation plasticity: to avoid “locking”, reduced (tangent) shear modulus is used to inform the stabilization stiffness [2]
- But reducing the stabilization stiffness too much can lead to hourglassing
- **Viscous** stabilization typically needed to supplement stiffness stabilization

[2] S. Reese. On a consistent hourglass stabilization technique to treat large inelastic deformations and thermo-mechanical coupling in plane strain problems. International journal for numerical methods in engineering, 57(8):1095-1127, 2003.

Existing approaches to develop an "elasto-plastic" form of hourglass control are either:

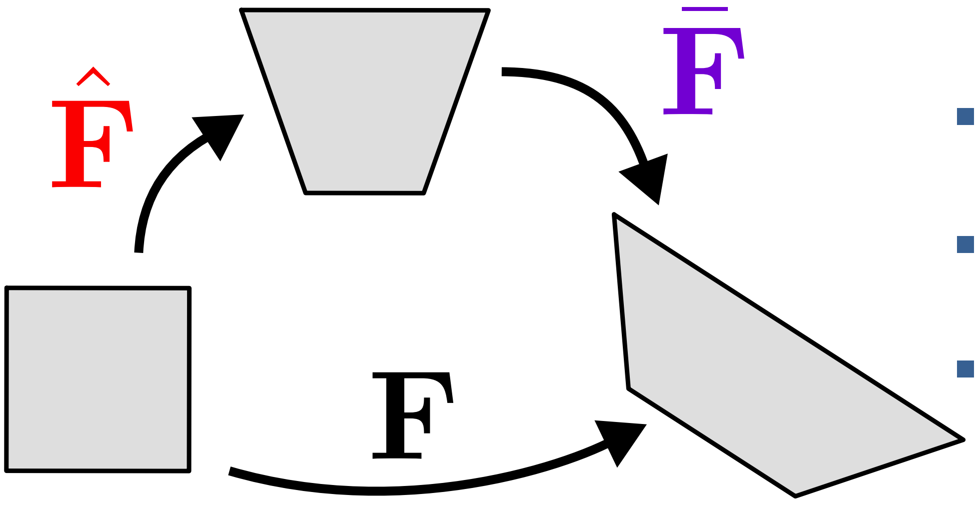
1. Not energetically consistent (may generate energy) [3]
2. Limited to specialized constitutive formulations [4,5]

Desired improvements for "elasto-plastic" hourglass control:

- Formulate within a thermodynamically consistent framework
- Decouple hourglass state from the constitutive state
- Minimize need for problem-specific adjustments

[3] L. Stainier and J.Ph. Ponthot. An improved one-point integration method for large strain elastoplastic analysis. Computer Methods in Applied Mechanics and Engineering, 118(1):163-177, 1994.
[4] P.H. Jetteur and S. Cescotto. A mixed finite element for the analysis of large inelastic strains. International Journal for Numerical Methods in Engineering, 31(2):229-239, 1991.
[5] X. Li, S. Cescotto, and P.G. Duxbury. A mixed strain element method for pressure-dependent elastoplasticity at moderate finite strain. International journal for numerical methods in engineering, 43(1):111-129, 1998.

Physical stabilization: decompose deformation gradient into average and hourglass parts



- Total deformation: $\boldsymbol{F} = \bar{\boldsymbol{F}}\hat{\boldsymbol{F}}$
- Hourglass deformation: $\hat{\boldsymbol{F}}$
- Average deformation: $\bar{\boldsymbol{F}}$

- Total energy: $\psi = \hat{\psi} + \bar{\psi}$
- Hourglass energy: $\hat{\psi}(\hat{\boldsymbol{F}})$
- Affine energy: $\bar{\psi}(\bar{\boldsymbol{F}})$

Stabilization stiffness

$$\boldsymbol{f} = \bar{\boldsymbol{K}}\bar{\boldsymbol{u}} + \hat{\boldsymbol{K}}\hat{\boldsymbol{u}}$$

Modal hourglass displacements

Co-rotational kinematics facilitates linearization & exact integration of stabilization stiffness

$$\hat{\boldsymbol{K}} = \int \hat{\boldsymbol{B}}^T \hat{\boldsymbol{D}} \hat{\boldsymbol{B}} dV$$

- Linearized co-rotational hourglass strains: $\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{B}}\hat{\boldsymbol{u}}$
- Linearized hourglass strain-displacement operator: $\hat{\boldsymbol{B}}$
- (Elastic) material stiffness: $\hat{\boldsymbol{D}}$

Postulate additive decomposition of hourglass displacements into elastic and plastic parts:

$$\hat{\boldsymbol{u}} = \hat{\boldsymbol{u}}^e + \hat{\boldsymbol{u}}^p$$

$$\hat{\boldsymbol{\varepsilon}} = \hat{\boldsymbol{\varepsilon}}^e + \hat{\boldsymbol{\varepsilon}}^p$$

$$\hat{\boldsymbol{\varepsilon}}^e = \widehat{\boldsymbol{B}}\hat{\boldsymbol{u}}^e \qquad \hat{\boldsymbol{\varepsilon}}^p = \widehat{\boldsymbol{B}}\hat{\boldsymbol{u}}^p$$

- Hourglass forces/stresses depend upon elastic hourglass displacements/strains:

$$\hat{\boldsymbol{f}} = \widehat{\boldsymbol{K}}\hat{\boldsymbol{u}}^e \qquad \hat{\boldsymbol{\sigma}} = \widehat{\boldsymbol{D}}\hat{\boldsymbol{\varepsilon}}^e$$

Define a non-local yield condition based on an integral measure of effective hourglass stress

$$f_y = \bar{\sigma} - \sigma_y \leq 0 \qquad \sigma_y = \sigma_0 + k_p \bar{\varepsilon}^p$$

$$\bar{\sigma} \equiv \sqrt{\frac{\int \hat{\boldsymbol{\sigma}} : \widehat{\mathbb{M}} : \hat{\boldsymbol{\sigma}}\, dV}{\int dV}} = \sqrt{\hat{\boldsymbol{u}}^e \widehat{\boldsymbol{M}} \hat{\boldsymbol{u}}^e}$$

- Hourglass yield condition $f_y \leq 0$ depends exclusively upon the hourglass deformations
- Hourglass plastic metric $\widehat{\boldsymbol{M}}$ can be integrated exactly

Flow rule for plastic hourglass displacements formulated to satisfy dissipation inequality

$$\mathcal{D}^p = \int \bar{\sigma} \dot{\varepsilon}^p dV = \hat{\boldsymbol{f}} \cdot \dot{\hat{\boldsymbol{u}}}^p \geq 0$$

$$\dot{\hat{\boldsymbol{u}}}^p = \hat{\boldsymbol{n}} \dot{\varepsilon}^p$$

- "Radial" flow direction with $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{u}}^e$ co-linear guarantees $\mathcal{D}^p \geq 0$

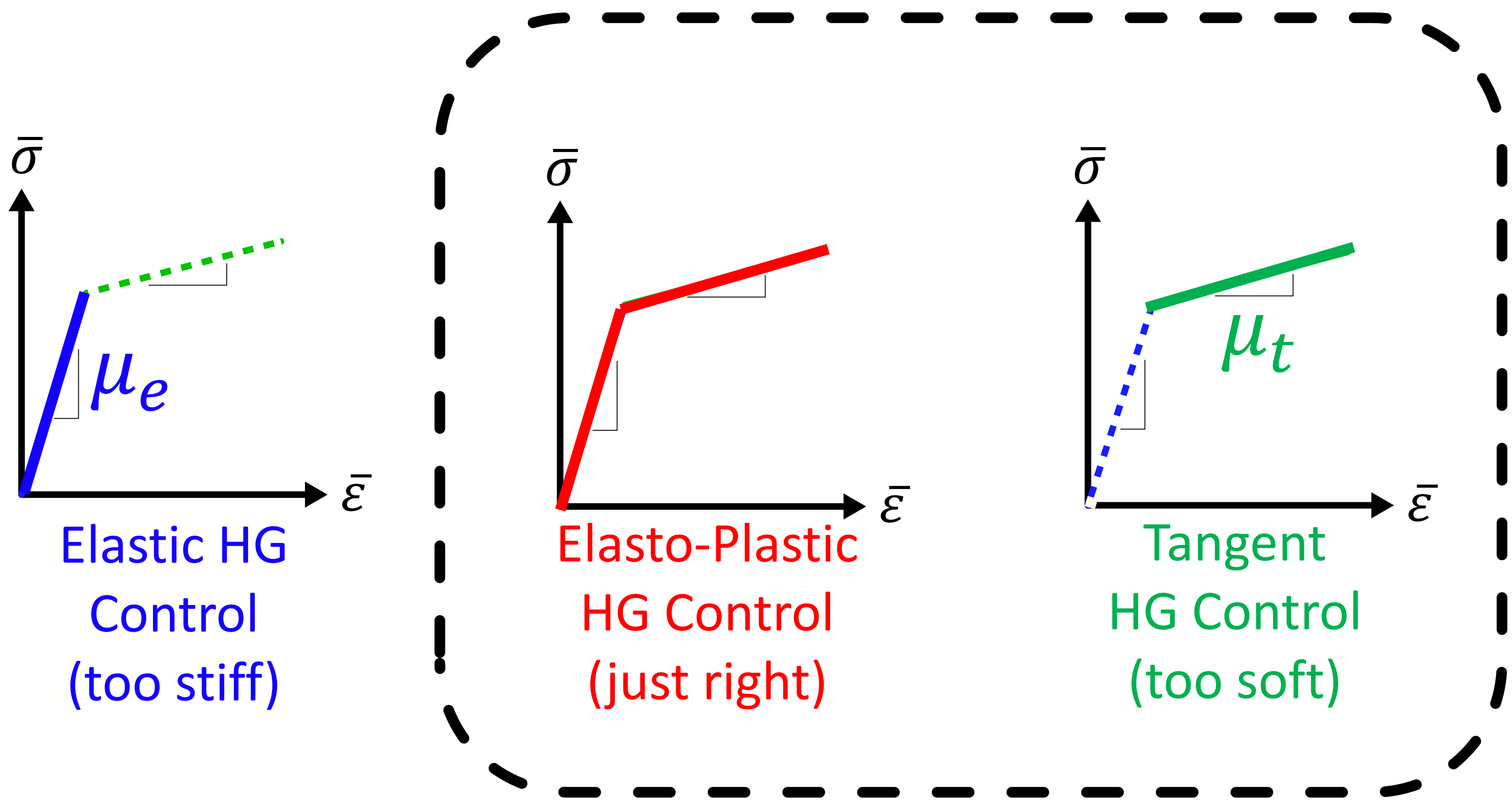
$$\hat{\boldsymbol{n}} = \frac{\int \bar{\sigma} dV}{\hat{\boldsymbol{u}}^e \hat{\boldsymbol{K}} \hat{\boldsymbol{u}}^e} \hat{\boldsymbol{u}}^e$$

Elasto-plastic predictor-corrector algorithm used to update the plastic hourglass displacements

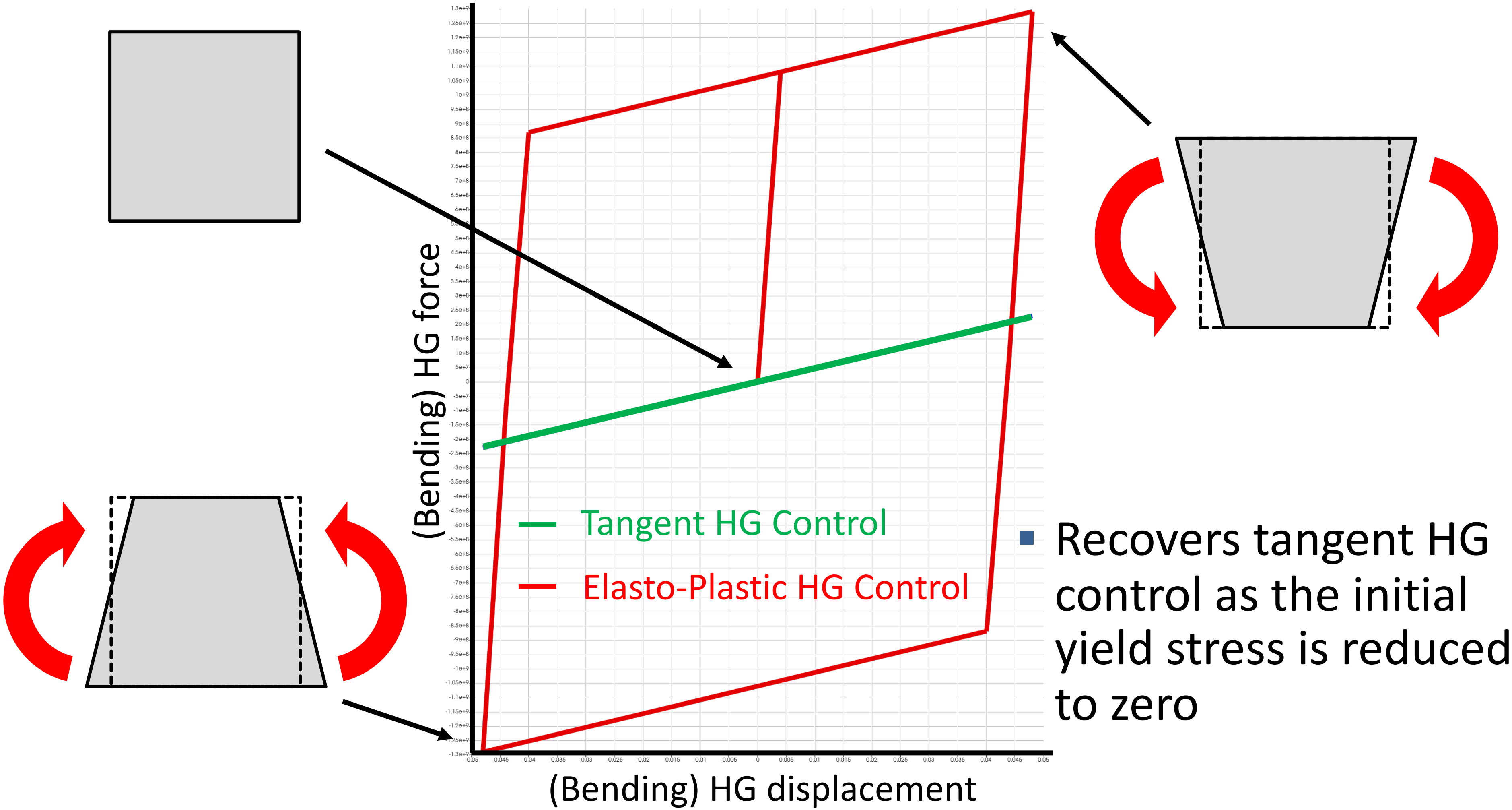
1. $\hat{\boldsymbol{u}}^{e,tr} = \hat{\boldsymbol{u}} - \hat{\boldsymbol{u}}^{p,tr}$
2. $\bar{\sigma}^{tr} = \sqrt{\hat{\boldsymbol{u}}^{e,tr}\widehat{\boldsymbol{M}}\hat{\boldsymbol{u}}^{e,tr}}$
3. $\|\hat{\boldsymbol{u}}^{e,tr}\| = \dfrac{\hat{\boldsymbol{u}}^{e,tr}\widehat{\boldsymbol{K}}\hat{\boldsymbol{u}}^{e,tr}}{\int \bar{\sigma}^{tr}\, dV}$
4. $\Delta\bar{\varepsilon}^{p} = \dfrac{\langle f_y^{tr}\rangle}{k_p + \bar{\sigma}^{tr}/\|\hat{\boldsymbol{u}}^{e,tr}\|}$
5. $\bar{\varepsilon}^{p} \leftarrow \bar{\varepsilon}^{p,tr} + \Delta\bar{\varepsilon}^{p}$
6. $\hat{\boldsymbol{u}}^{p} \leftarrow \hat{\boldsymbol{u}}^{p,tr} + \dfrac{\hat{\boldsymbol{u}}^{e,tr}}{\|\hat{\boldsymbol{u}}^{e,tr}\|}\Delta\bar{\varepsilon}^{p}$
7. $\hat{\boldsymbol{f}} = \widehat{\boldsymbol{K}}(\hat{\boldsymbol{u}} - \hat{\boldsymbol{u}}^{p})$

- Elastic predictor step:
  - $\dot{\hat{\boldsymbol{u}}} \neq 0,\ \dot{\hat{\boldsymbol{u}}}^{p} = 0$
- Plastic corrector step:
  - $\dot{\hat{\boldsymbol{u}}} = 0,\ \dot{\hat{\boldsymbol{u}}}^{p} \neq 0$, radial return $\hat{\boldsymbol{n}}$
- Negligible added computational cost to original hourglass force computations
- Requires storage of (12) plastic hourglass displacements, and equivalent plastic strain

Demonstrations and comparison between elasto-plastic vs. elastic/tangent HG control

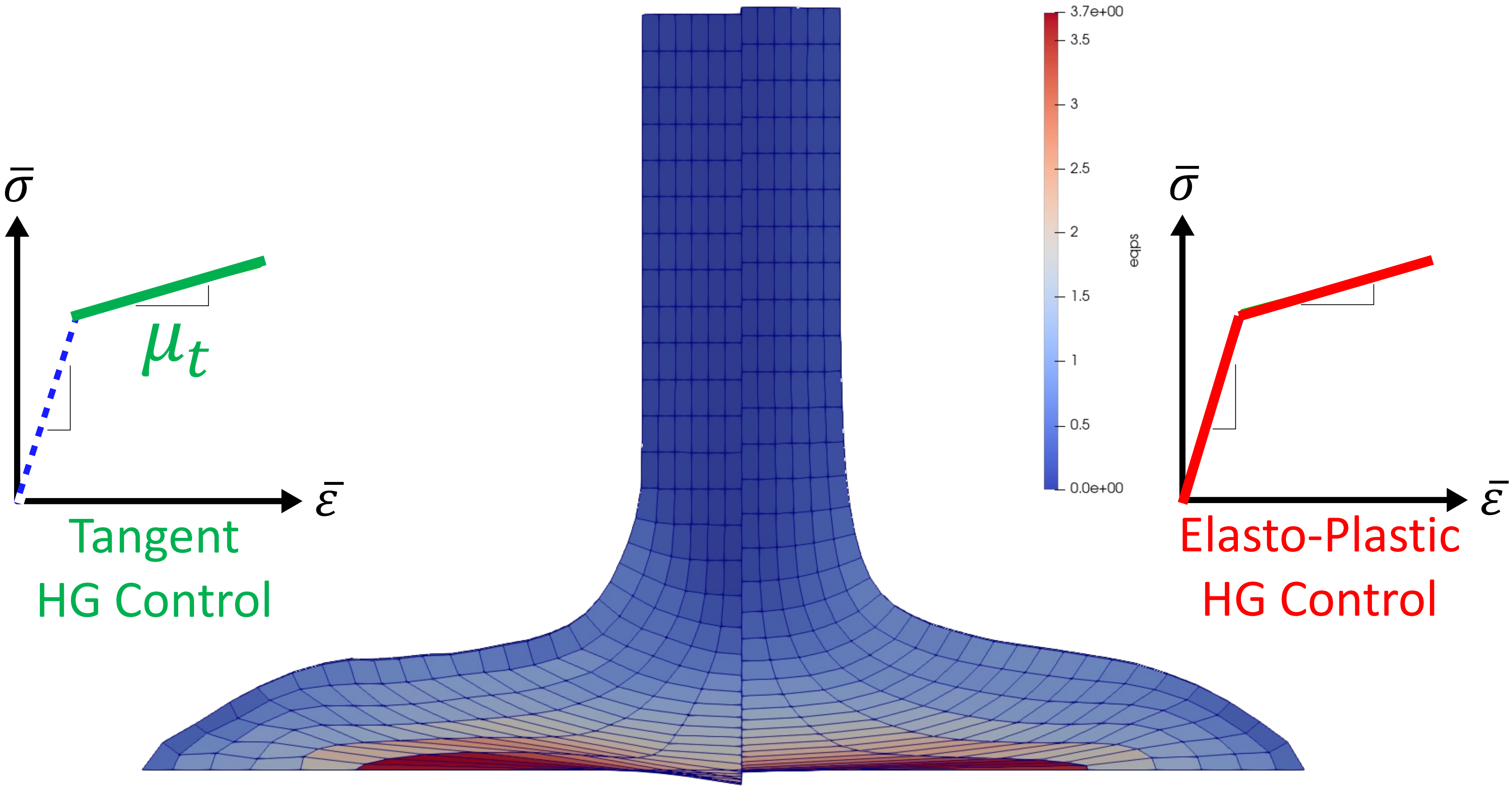


Single element example problem: cyclic bending with kinematic hardening behavior



- Recovers tangent HG control as the initial yield stress is reduced to zero

Elasto-plastic HG control avoids both locking (too stiff) and hourglassing (too soft) behaviors

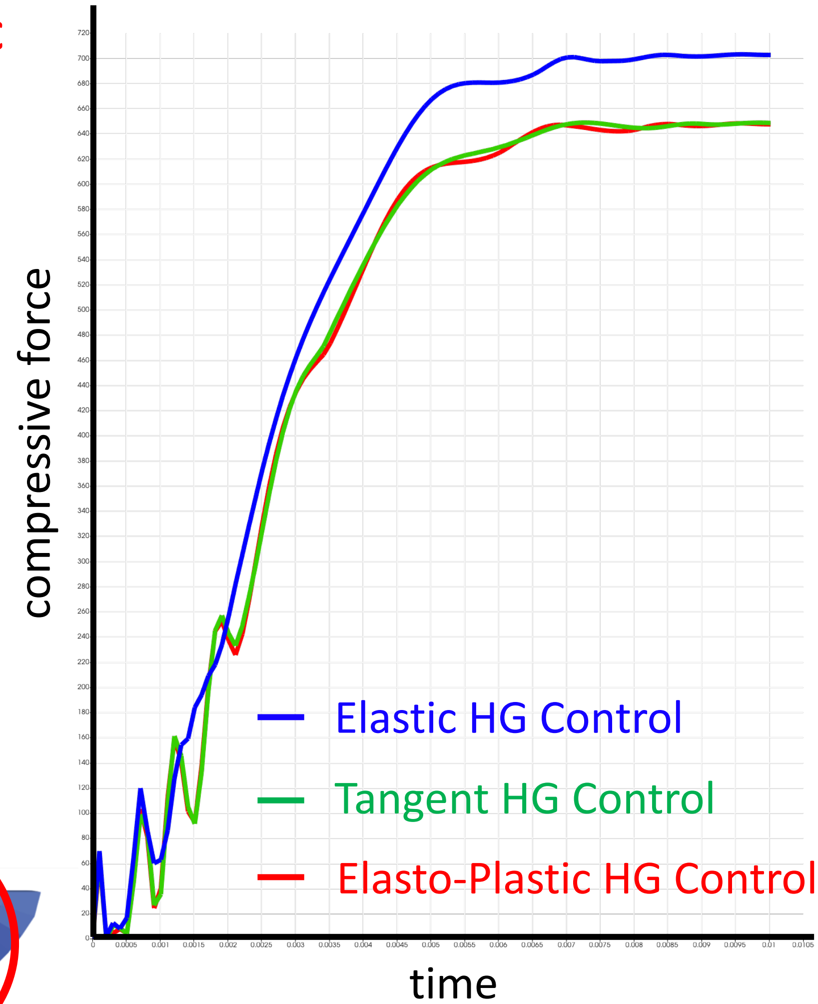


Tangent
HG Control

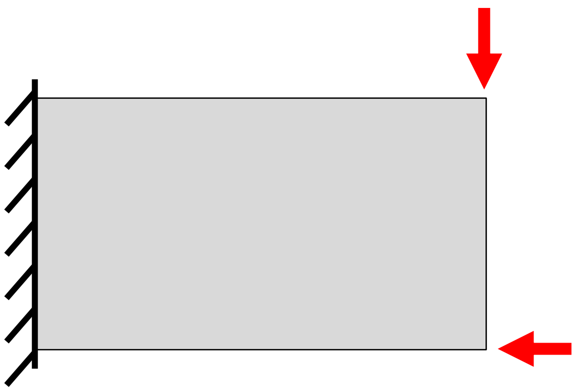
Elasto-Plastic
HG Control

Compressed aluminum billet: deformations and net force compare well with tangent HG control

- Plotted hourglass plastic strains demonstrate localization in *highly distorted elements*

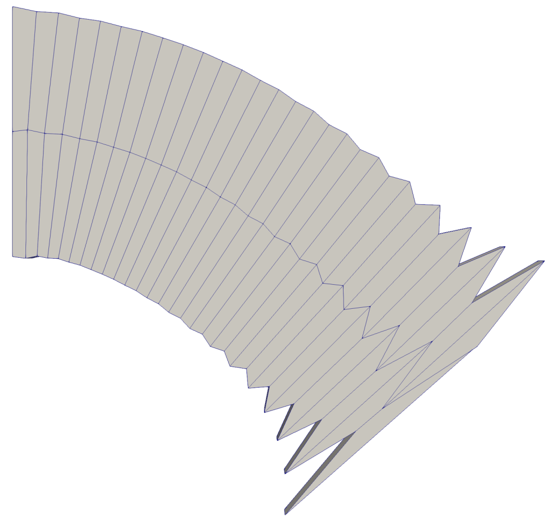


Concentrated loading excites low-energy hourglass modes, even under mesh refinement

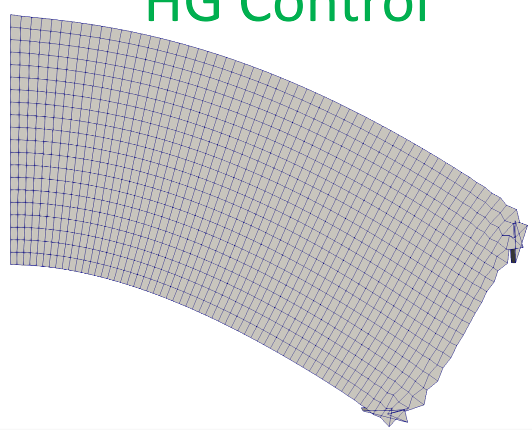


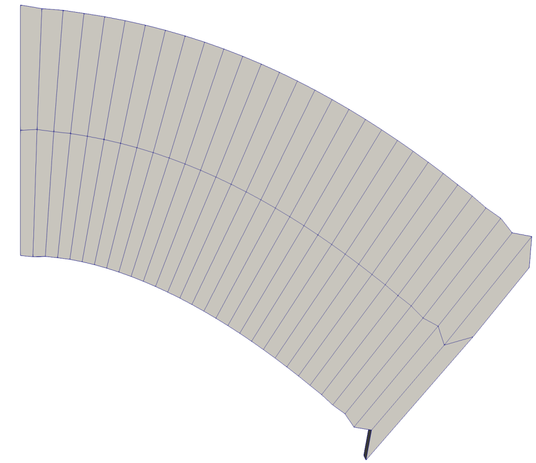
- Coarsely meshed beam bending problem from [6]
- Resulting behavior is **purely elastic** (no yielding)

[6] Y. Ko and K.-J. Bathe. A new 8-node element for analysis of three-dimensional solids. Computers & Structures, 202:85-104, 2018. ISSN 0045-7949.



Tangent
HG Control

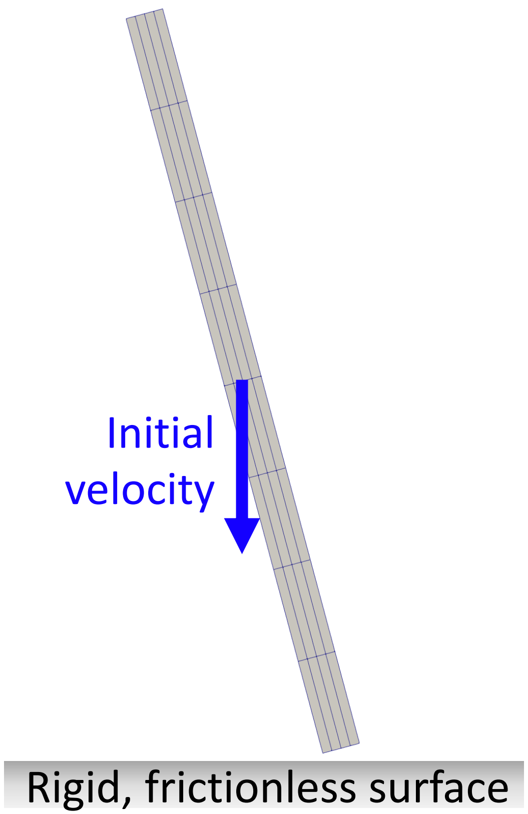




Elasto-Plastic
HG Control

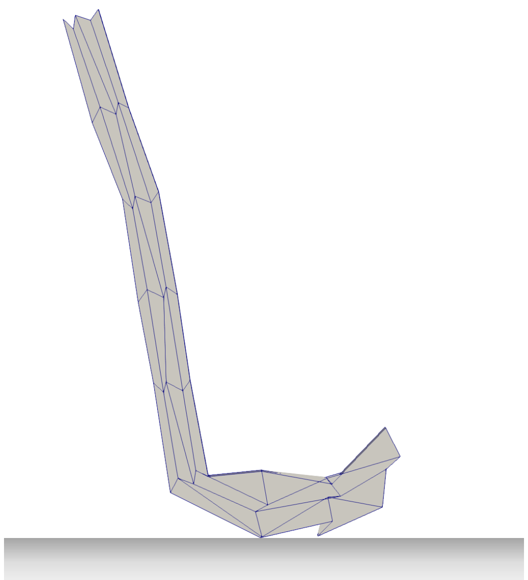
- Good coarse-mesh bending accuracy using EAS modes in the HG control [1]

Concentrated loading from contact forces in high-velocity impact exaggerates hourglassing



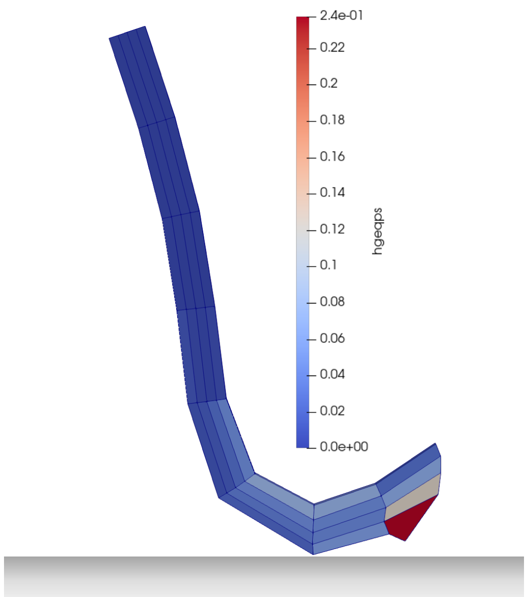
- Thin geometries are commonly meshed anisotropically

Tangent HG Control



- Contact force at bottom corner node excites hourglassing

Elasto-Plastic HG Control



- Plot of localized hourglass plastic strains
- No locking or hourglassing

Conclusions & future work

- Proposed methodology provides several improvements:
  - Avoids locking (too stiff) and hourglassing (too soft) behaviors
  - Plastic dissipation obviates need for viscous hourglass control
  - Stabilization parameters informed directly by the plasticity model
    - Avoids heuristic adjustments to the stabilization stiffness
- Polyhedral element technologies (VEM) require stabilization in both explicit & implicit analyses
  - Poor selection of the stabilizing parameters can lead to ill-conditioning
  - **Future work**: apply proposed methodology to VEM
- Composite/super-elements
  - **Future work**: extend approach for non-linear reduced order modeling

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